GCSE Maths – Geometry and Measures

Trigonometric Ratios and Exact Trigonometric Values

Notes

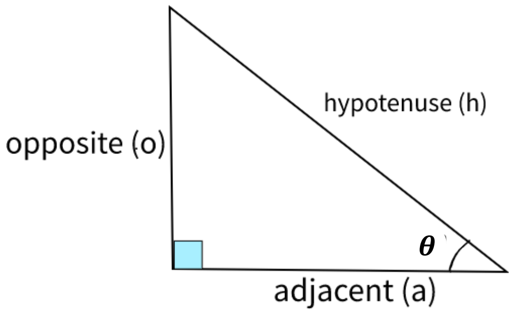
WORKSHEET

This work by PMT Education is licensed under CC BY-NC-ND 4.0

Trigonometric Ratios and Exact Trig Values

Labelling a right-angled triangle

When working with **right-angled triangles**, there are three important ratios between the sides and angles. Before we can understand these, you must be able to label the right-angled triangle correctly.



The **hypotenuse**, denoted by the letter h, is the longest side of the triangle, opposite the right-angle.

The **opposite side**, o, is the side opposite the angle θ which we are working with.

The **adjacent side**, a, is the one adjacent to the angle we are working with, that is not the hypotenuse.

Trigonometric Ratios

The three trigonometric ratios you must know are for sine, cosine and tangent:

$$\sin\theta = \frac{opposite}{hypotenuse}$$

$$\cos\theta = \frac{adjacent}{hypotenuse}$$

$$\tan\theta = \frac{opposite}{adjacent}$$

You can remember these ratios by the phrase **SOH CAH TOA**.
The first letter indicates the trigonometric ratio and the second letter represents the length which is divided by the length represented by the third letter.

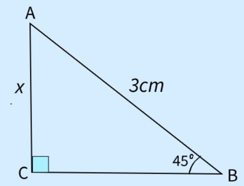
For example, SOH helps us to remember that

$$Sin\ \theta = \frac{Opposite}{Hypotenuse}.$$

Using the trigonometric ratios, you can work out an unknown side or unknown angle in a right-angled triangle.

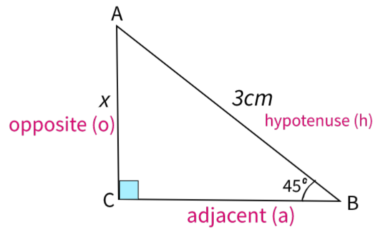
CC BY NC ND

Example: Work out the length of side x in the right-angled triangle ABC, to 3 significant figures.



1. **Label the sides** of the triangle according to the known angle and decide which **trigonometric ratio** to use depending on which sides we know.

   Here, we know the length of the hypotenuse, and are trying to find the opposite side. So, we use the sine ratio:



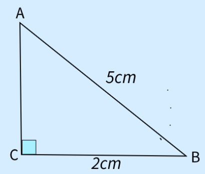
$$\sin\theta = \frac{opposite}{hypotenuse}$$

2. **Substitute** our known values into the trigonometric ratio and **solve** the equation.

$$\sin 45 = \frac{x}{3}$$

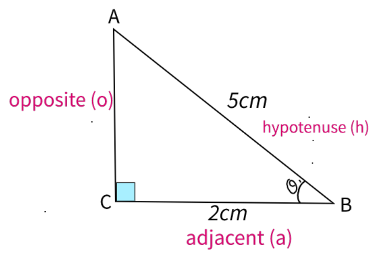
$$x = 3 \times \sin 45 = \mathbf{2.12\ cm}$$

Example: Find the size of angle ABC in the triangle below, to the nearest degree.



1. **Label the sides** of the triangle according to the unknown angle and decide which trigonometric ratio to use depending on which sides we know.

   We know the lengths of the adjacent side and hypotenuse, so we know to use the cosine ratio:



$$\cos\theta = \frac{adjacent}{hypotenuse}$$

2. **Substitute** our known values into the cosine ratio.

$$cos\ \theta = \frac{adjacent}{hypotenuse} = \frac{2}{5}$$

3. In this example, we are finding an unknown angle. So, we are required to use the **inverse function**, for which there is a button on the calculator.

$$cos\ \theta = \frac{2}{5}$$

$$\theta = cos^{-1}\left(\frac{2}{5}\right) = 66.4218\ldots° = \mathbf{66}°$$ (to the nearest degree)

CC BY NC ND

Exact trigonometric values

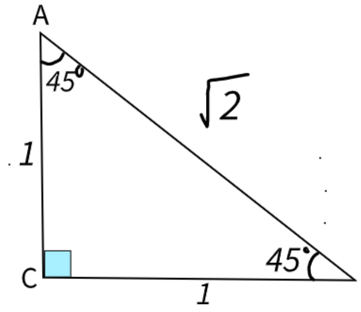
You need to be able to find, without a calculator, the exact values of $\sin\theta$ and $\cos\theta$ for $\theta = 0°, 30°, 45°, 60°$ and $90°$, and the exact values for $\tan\theta$ for $\theta = 0°, 30°, 45°$ and $60°$.

The exact values you need to know are summarised in the table below:

$\theta =$	0	$30°$	$45°$	$60°$	$90°$
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	-

You don't need to memorise the three middle columns related to $\theta = 30°, 45°, 60°$ as you can construct the exact values from the following special triangles.

Isosceles right-angled triangle, with the two sides of equal length measuring 1 unit, two remaining angles measuring $45°$:

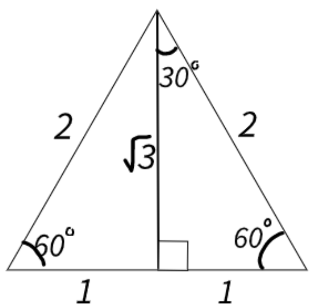


- We can work out the length of the hypotenuse using Pythagoras' Theorem:

$$hypotenuse^2 = 1^2 + 1^2$$
$$hypotenuse^2 = 2$$
$$hypotenuse = \sqrt{2}$$

The trigonometric exact values when $\theta = 45°$ are then calculated using trigonometric ratios. For example, to find $\cos 45$, since $\cos\theta = \frac{adjacent}{hypotenuse}$, we have $\cos 45 = \frac{1}{\sqrt{2}}$.

Equilateral triangle, with side lengths measuring 2 units, angles measuring $60°$:



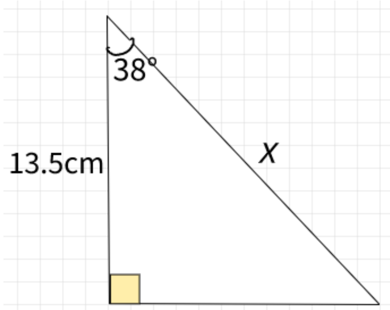
- We draw a perpendicular bisector down the middle to form a right-angled triangle with angles measuring $30°$, $60°$ and $90°$.
- We can work out the length of the bisector using Pythagoras' Theorem:

$$bisector^2 = 2^2 - 1^2$$
$$bisector^2 = 3$$
$$bisector = \sqrt{3}$$

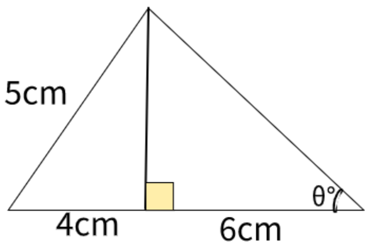
CC BY NC ND

Trigonometric Ratios and Exact Trig Values – Practice Questions

1. Find the length of side x to 1 decimal place.



2. Find the size of angle θ to 1 decimal place.

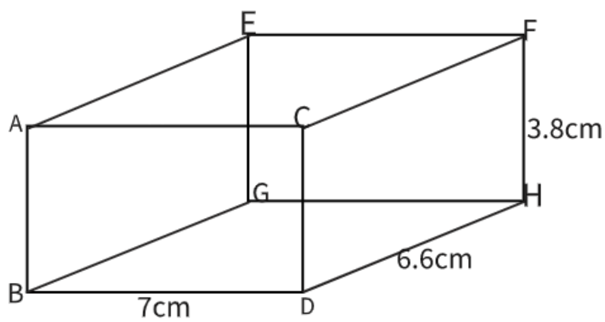


3. Find the exact value of $\sin 30°$.

4. Find the exact value of $\tan 45°$.

5. (Higher only) In the following cuboid, find to 3 significant figures:

   a) the length of side BH
   b) the value of angle BHF



Worked solutions for the practice questions can be found amongst the worked solutions for the corresponding worksheet file.